THE MOVEMENTS OF TROUGHS AND RIDGES ON THE 500-MB CONTINUITY CHART

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by

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PREFACE

The present study of the Continuity Chart was undertaken at the suggestion of LCDR Paul Wolff, USN, as part of the work being done by Project Arowa in their studies of the 500-mb surface. It was done at the United States Naval Postgraduate School, Monterey, California, during the spring of 1954.

My thanks are tendered to LCDR Wolff for furnishing the 5-day mean 500-mb charts and corresponding continuity charts, as well as for his original suggestions which led to this study; to Professor W. D. Duthie, for his continuous guidance and for suggesting several interesting and fruitful approaches to the problem; to Professor C. L. Perry, for his unflagging patience in instructing a non-mechanically minded person in the use of the mathematical machines without which this study would not have been possible, as well as for pointing the way toward the derivation of several of the equations in the text; to CDR Ernest E. Butow, USNR, for his help in much of the tedious work of harmonic analysis; to Sidney Freshour, for assistance in wiring and operating the Analogue Computer; and last but not least to my wife, Theresa, for her invaluable assistance in the typing of the manuscript.

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CHAPTER I

INTRODUCTION

1. Defining a Wave.

Since the formal statement of the wave theory of cyclones by V. Bjerknes [2], a vast literature has evolved studying the wave motion in the atmosphere under various restricting qualifications. It would be a complete thesis in itself to compile a bibliography of this work, and much of it would be irrelevent to the present study. The Compendium of Meteorology [9] contains a recent partial review of the more significant publications in this field.

Even before the pioneering work of the Norwegian school of meteorology, wave phenomena have engaged the work of physicists in many fields. The work of Newton, Laplace, Poisson, Helmholtz, Stokes, Rossby, Haurwitz, and others has been recently reviewed by Queney [13]. Outside the realm of physics, the references to waves approach the infinite. With this vast background, the question of "what is a wave" may appear either trivial or obvious, but simply because the word has been used so much by both technicians and lay people, it has lost any precise meaning that it might once have had, and a definition is vitally needed.

The mathematician would define a wave as any function that satisfies the equation

$$\frac{\partial^2 f}{\partial t^2} = c^2 \nabla^2 f$$

where f is any function, c is a constant, and ∇^2 is the Laplacian operator. However this appears to be too restrictive for many

meteorological applications.

The Meteorological Glossary [10] gives these definitions of a wave:

> Any regular periodic oscillations, the most noticeable case being that of waves on the sea. . . . .
> Any periodic oscillation either of the air, water, temperature, or any other variable, recurring more or less regularly, may be referred to as waves. . . . .
> In America 'heat waves' and 'cold waves' are spoken of. These are spells of hot and cold weather without any definite duration, and do not recur regularly. . . . .

In other words, a wave can be a phenomenon which is neither regular nor periodic. This leaves only the word "oscillation" in the definition and though this may not appear to define very much, it appears to be what is meant by meteorologists in much of the literature referred to above.

Yet even this may be too restrictive, if by oscillation is meant a movement first on one side and then on the other of some determined normal or mean position. The heat waves and cold waves referred to by the Glossary are departures from normal but upon recovery of the normal value (of temperature) there is no compensatory movement to the other side of the mean. Perturbation seems to be the best word for this kind of wave, and will be used throughout this paper when a departure from the mean on one side only is meant.

2. Description of the Continuity Chart.

The difficulties sketched above in defining waves have led to the increasing use by meteorologists of the continuity chart. Although not named by him, this chart seems to have been introduced as a forecasting tool by Cressman [4]. He plotted daily charts of the

average height in feet of the 500-mb surface between 40N (latitude 40 degrees north) and 55N and used this sequence of graphs to demonstrate downstream propagation of energy. More recently, Riehl et al [14] have given an explicit description of these charts as follows:

> If the tabulated heights on a given latitude are plotted as a function of longitude and the resulting points joined with a smooth curve, we obtain a representation of the longitudinal variations in the height of the isobaric surface. By plotting such curves for successive days below one another, the time continuity of these longitudinal variations can be followed. The continuity chart is helpful in determining long wave positions and group velocity.

One of the implied assumptions in this statement by Riehl should be stated explicitly. It is that the motion of waves is primarily zonal. A perturbation of the 500-mb surface traveling along a latitude circle will have time continuity on a chart such as Riehl describes but a perturbation traveling across the latitude circle would appear as stationary with changing amplitude. Perhaps it is some of these perturbations with significant meridional components of motion that account for some of the discontinuities of the continuity chart. However, synoptic meteorological experience is that atmospheric motion is primarily zonal and the continuity chart has found favor as a forecasting tool.

Recently Project Arowa has prepared such continuity charts as part of their 500-mb studies [1]. An example is shown in Plate I. The continuity charts made by them are prepared from the 5-day mean 500-mb charts constructed at the same activity. These mean charts are not prepared at intervals of five days as at the long-range forecast center of the United States Weather Bureau [12], but are made each day. There is thus no question of whether a trough moves

forward (eastward) or retrogresses in the interval between maps. From the 5-day mean 500-mb charts the height of the surface around the 50N latitude circle is read at 10-degree intervals of longitude and those heights connected by a smooth curve. This is the identical method advocated by Riehl except that the Arowa curves cover the entire latitude circle.

This paper is an investigation of the properties of the waves or perturbations shown on such continuity charts, with a view to increasing the usefulness of the charts for forecasting purposes, including the prognosis of the chart itself.

3. Summary and Conclusions.

Chapter II explains the derivation of the parameters of perturbation length, amplitude, and steepness, and correlates these with the perturbation velocities. The investigation reveals that neither the primary parameters nor their changes with time show sufficient correlations with velocity to be useful as prognostic tools.

Chapter III discusses the applications of a restricted type of harmonic analysis to the continuity chart. A formula for perturbation velocity is derived and a hypothetical example of its use explained. The results of harmonic analysis of daily and 5-day mean continuity charts are presented and compared. It is concluded that harmonic analysis can be a useful forecasting tool for some purposes and that it is more useful for the 5-day mean charts than for the daily charts. It is believed that it might be even more useful for space-mean charts.

CHAPTER II

THE MOTION OF PERTURBATIONS ON 5-DAY MEAN CONTINUITY CHARTS

1. Determination of Wave Length.

Plate I is a sample of the continuity charts prepared by Project Arowa. On these charts the solid curve represents the 5-day mean height of the 500-mb surface at 50N around the entire hemisphere. Despite the averaging process involved in preparing these charts the resulting curves are far from simple. It would appear that the obvious way to define wave length would be to measure the distance between successive maxima or minima. However there is frequently a relative minimum between two maxima, as illustrated by the curves for September 1 and 2 on Plate I where this occurs between 10W and 90E. This configuration appears to be the result of the superposition of two waves of different lengths and amplitudes, and has been assumed to be such by several writers, notably Riehl [14]. The short waves are supposed to be damped out by the 5-day averaging process and in fact this is one of the reasons given by Rossby [15] for the construction of 5-day mean charts. If this is true, all of the waves shown on the Arowa continuity charts should be long waves.

In an attempt to cover this difficulty the Arowa workers have superimposed on their continuity charts monthly mean-height curves derived from the Weather Bureau Normal Charts [17]. These monthly mean curves are shown as dashed lines on Plate I. By comparison of these two curves it is possible to tell at a glance whether the 500-mb surface for a particular 5-day period is above or below

normal at any longitude. It was felt by LCDR Wolff* that the normal curve represented the standing perturbation of the atmosphere, such as described by Bolin [3] and ascribed by him to orographic features. The departure of the 5-day mean surface from the normal would then serve to delineate the moving and changing waves from the more or less permanent standing waves. Since it is the departures of the pattern from the normal that interest the forecaster, subtraction of the standing perturbation from the flow should serve to simplify the problem of determining the wave length. Unfortunately this sanguine expectation was not realized, since occasionally the 500-mb height is below normal around the entire latitude circle. Since a wave length of greater than 360 degrees has no meaning in meteorology, the departure from normal had to be abandoned as a tool for defining wave length.

The failure of the long-term mean (i.e. normals) to delineate waves objectively on the continuity charts naturally suggested determining the mean over a shorter period. If the heights of the 500-mb surface at each 10 degrees of longitude are averaged for each chart individually, a 5-day mean is determined. Ideally, this would divide the curve into an equal number of troughs and ridges. The wave length could be defined as the distance between successive crests or troughs and the amplitude as the maximum departure of the curve from its own mean. Accordingly the means were computed for each 5-day period and added to the continuity charts. These are shown as the solid horizontal lines on Plate I. Inspection of Plate I shows that the situation is still not simple. The curve may remain above or below its own mean for distances up to 180 degrees of

*Personal communication

longitude, with several extreme values, so that it is very difficult to define what it the crest or trough longitude and hence to determine a wave length. This is illustrated on Plate I by the curves for September 14, between 70W and 110E and for September 15, between 60W and 110E. Furthermore, the ridge and trough portions of the curve are far from symmetric. Ridges of small amplitude can be followed and preceded by troughs of great amplitude and vice versa. Hence only an average amplitude can be defined for a complete oscillation. Obviously the waves shown on the continuity charts are not "regular periodic oscillations".

For these reasons it was decided to treat ridges and troughs separately. From the point at which the continuity curve crossed its own mean to their next intersection was considered to be a wedge or trough, depending on whether the curve went above or below its own mean. However even this simplification was not sufficient, as frequently the continuity curve would approach the mean very closely from above or below and then depart again on the same side as that from which it approached. This situation is shown on Plate I, by the curve for September 14, where it occurs at 40W and again at 72E. It was felt that it would be arbitrary to the point of meaninglessness to insist that the height curve should actually touch the mean when it is obvious that these are cases of multiple perturbations. It was therefore decided in these cases to regard the mean as a zone 200 feet wide (± 100 feet from the arithmetic mean). If the height curve came within this zone, this was regarded as the end of one perturbation, and when it again emerged from the zone as the start of another. It is thus possible to have two wedges (or troughs) in

succession, and it is also possible that the sum of all wedge and trough lengths for any particular day will not equal 360 degrees. The mean zone as distinct from the arithmetic mean was only used when the height curve approached very closely to the arithmetic mean and then deviated again on the same side. The arithmetic mean was always used to define the amplitudes of the perturbations. The use of the mean zone is shown on Plate I where it has been added to the curve for September 7, and is shown as a shaded area. On this curve the use of a mean zone serves to break the ridge extending from 20W to 100E into two ridges, one from 20W to 45E and one from 60E to 100E.

2. Frequency Distributions of Perturbation Parameters.

The frequency distribution of perturbation lengths is shown in Table 1.

This is obviously a very skewed distribution with the mean widely separated from the middle frequency. Furthur inspection of the charts showed that in almost all cases perturbation lengths greater than 90 degrees consisted of two or more perturbations merging into each other, but that the minimum amplitude separating them did not reach to the mean zone defined above. It was felt that the inclusion of these very long and questionable lengths might vitiate any subsequent results. For this reason all lengths greater than 90 degrees were eliminated from further consideration. This resulted in a total of 94 wedges and 114 troughs which were considered in subsequent calculations. Table 1 does not show any apparent differences in the distributions of wedges and troughs.

TABLE 1

Frequency Distribution of Perturbation Lengths
(measured in degrees longitude of 50N)

Length	Wedges	Troughs	Total
0 - 9	1	2	3
10 - 19	6	7	13
20 - 29	11	20	31
30 - 39	25	19	44
40 - 49	17	23	40
50 - 59	18	18	36
60 - 69	7	12	19
70 - 79	5	6	11
80 - 89	4	7	11
90 - 99	2	9	11
100 - 109	1	6	7
110 - 119	7	1	8
120 - 129	2	2	4
130 - 139	0	3	3
140 - 149	0	1	1
150 - 159	0	0	0
160 - 169	0	1	1
170 - 179	2	0	2
Total	108	137	245

The next step was the classification of perturbations by amplitudes. This is shown in Table 2. Once again there are no apparent differences in distributions.

TABLE 2

Frequency Distribution of Perturbation Amplitudes
(measured in 10's of feet)

Amplitude	Wedges	Troughs	Total
0 - 9	9	9	18
10 - 19	13	12	25
20 - 29	14	22	36
30 - 39	15	22	37
40 - 49	14	24	38
50 - 59	21	17	38
60 - 69	2	6	8
70 - 79	2	1	3
80 - 89	2	1	3
90 - 99	2	0	2
Total	94	114	208

As shown by Lamb [7] for so-called Stokes waves, the steepness is a valuable parameter. This is defined as the wave amplitude divided by the wave length. In order to determine if steepness had any relation to the velocities of the perturbations shown on the continuity chart, the steepness was computed for each perturbation. The units of steepness are nondimensional and for the purposes of this paper only a relative measure was considered necessary. Accordingly amplitude and length were not expressed in the same units,

so that the units of steepness are entirely arbitrary, and valid only for comparison purposes. The distribution of steepnesses is shown in Table 3. Again there are no apparent differences in distribution.

TABLE 3

Frequency Distribution of Perturbation Steepnesses
(measured in arbitrary units)

Steepness	Wedges	Troughs	Total
0 - 9	2	0	2
10 - 19	9	8	17
20 - 29	15	20	35
30 - 39	22	35	57
40 - 49	19	20	39
50 - 59	14	15	29
60 - 69	5	10	15
70 - 79	4	4	8
80 - 89	2	2	4
90 - 99	2	0	2
Total	94	114	208

3. Correlations between Perturbation Parameters.

In order to determine the extent of relationship of steepness to amplitude and perturbation length, the product-moment correlation coefficients were computed. These results are shown in Table 4.

It is apparent from these correlations that length and amplitude are not independent variables. The correlation between these parameters is for wedges 0.709 and for troughs 0.687. This shows the not too surprising fact that long perturbations tend to have greater

amplitudes than short ones. The high correlation between amplitude and steepness indicates that the additional parameter of steepness adds little to the description of a perturbation that is not already shown by length and steepness taken together or separately. Nevertheless, various computations were undertaken with this parameter as discussed in the next section.

TABLE 4

Correlation Coefficients between Various Pairs of Parameters

Correlation between	Wedges	Troughs
length-steepness	.149	-.181
amplitude-steepness	.803	.591

4. Movement of Perturbations.

When continuity charts are regarded in sequence it is frequently possible to make a tentative identification of wedges and troughs from one day to the next. It is probably for this reason that they have been called continuity charts, for they frequently seem to show a continuous movement of wedges and troughs at varying rates of speed. It is possible to define this speed in various ways. One could take the longitude of the maximum amplitude and consider its day-to-day change as an average 24-hour speed. However, in the case of more or less flat troughs and ridges, the longitude of maximum amplitude may be difficult to determine objectively. In keeping therefore with the empirical and objective nature of this study, it was decided to define the longitude of a perturbation as the longitude midway between its beginning and end. These points are objectively determinable as shown previously. Having determined a

mean longitude (or longitude of the mid-point), the speed of the perturbation can then be defined in a completely objective fashion as the 24-hour movement of this mean longitude.

Examination of the continuity charts for September 1951 showed that it was possible to identify 60 of the previously delineated 84 wedges for periods up to 16 days and 93 of the 114 troughs for periods up to 23 days. Undoubtedly some of these perturbations could have been identified for even longer periods, but time limitations precluded the use of a larger sample. The velocity distribution is shown in Table 5.

TABLE 5

Frequency Distribution of Perturbation Velocities

	Velocity (degrees longitude at 50N /day)	Wedges	Troughs	Total
	19-17	0	0	0
westward	16-14	0	2	2
	13-11	1	0	1
	10-8	1	3	4
	7-5	1	9	10
	4-2	3	13	16
—	1-1	17	19	36
	2-4	22	19	41
eastward	5-7	5	12	17
	8-10	2	4	6
	11-13	0	3	3
	14-16	2	0	2
	17-19	0	1	1
	Total	54	85	139

With a velocity defined, it is possible to compare it to wave length, amplitude, and steepness. Once again the product moment correlation coefficient appears to be the proper tool, so these were computed as shown in Table 6.

TABLE 6

Correlation Coefficients between Velocities and Primary Parameters

Correlation of Velocity	Wedges	Troughs
with amplitude	-.235	-.081
with length	-.260	-.220
with steepness	.0001	.0000

Although all these correlations are very small and of absolutely no value for prognostic purposes they do have the correct sign to agree with some of the ideas expressed in the literature. (See for instance Riehl [14]). Long perturbations show a slight tendency toward retrograde motion and since amplitudes are strongly correlated positively with lengths, the larger amplitudes also tend toward retrograde motion. It is also apparent that the steepness parameter as defined above, is of no prognostic value whatsoever.

As a final attempt to find something of prognostic value in the study, the changes of amplitude, length, and steepness were computed from day to day and correlations computed between these changes and velocities. These correlations are shown in Table 7.

Here again the relationships are so small as to be of no prognostic value. Since long perturbations tend to move slower it would be expected that as they get longer the velocity would be less and the same remark applies to the changes of amplitudes.

TABLE 7

Correlation Coefficients between Velocities and the Changes of Various Parameters

Correlation of velocity with change of	Wedges	Troughs
amplitude	-.186	-.170
length	-.358	-.132
steepness	.152	.140

5. Conclusions

In concluding this stage of the study at this point, it was felt that none of the computations justified further investigation of other months or other years. Although it is possible that better correlations might be found at other times, the fact that they are so poor at least once indicates that they may well be so again at some other time, and if they are ever as bad as shown in this admittedly small sample, they can hardly have any value as prognostic tools.

As a positive result it can be said that the strong relationship between amplitude and length of perturbations and between amplitude and steepness shows that in future studies only one of these parameters need be considered.

The failure of these parameters to qualify as prognostic tools led to an approach to the problem of forecasting the continuity chart by means of harmonic analysis. This is discussed in the next chapter.

CHAPTER III

HARMONIC ANALYSIS OF THE CONTINUITY CHART

Many of the wave equations in the meteorological literature referred to earlier make the assumption that the streamlines (or trajectories) are simple sine curves. (See for instance Rossby [15] or Holmboe [6]). This is obviously only a poor approximation at best, and yet it has resulted in some fruitful and useful prognostic tools. Since by the theory of Fourier analysis any function can be approximated to any desired degree of accuracy by a Fourier series of sine and cosine terms it seemed desirable to investigate the possibility of using harmonic analysis to determine the waves which might combine to form the 500-mb profile along a latitude circle, as well as the possibility of forecasting the movement of the Fourier components and then recombining them into a prognosis of the continuity chart.

1. Theoretical Considerations.

In deriving his well-known wave equation, Rossby [15] makes the following assumptions:

1. Inviscid, homogeneous, imcompressible atmosphere.
2. Purely horizontal motion
3. Constant absolute vorticity
4. Rate of change of Coriolis force is constant with latitude.

He then applies perturbation theory to the case of the simple sinusoidal perturbation. As the solution to the differential equation

$$\left[\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right]\left[\frac{\partial^2 \psi}{\partial x^2}\right] + \beta\frac{\partial \psi}{\partial x} = 0 \tag{1}$$

he assumes a stream function of the form

$$\psi = A \cos k(x - ct) , \tag{2}$$

where A is the amplitude, k is the wave number (equal to $\frac{2\pi}{L}$), c is the speed of movement of the wave form, β is the rate of change of the Coriolis parameter, and U is the basic (unperturbed) zonal current. Thence he derives the equation

$$c = U - \frac{\beta L^2}{4\pi^2} , \quad \text{or} \quad c = U\left(1 - \frac{L^2}{L_s^2}\right) . \tag{3}$$

Now let us consider a stream function of the form

$$\psi = \sum_{n=1}^{\infty} A_n \cos k_n(x - c_n t) , \tag{4}$$

which also satisfies the differential equation (1) as follows:

$$\begin{aligned}\sum_{n=1}^{\infty} & k_n^3 c_n A_n \sin k_n (x - c_n t) \\ & - U\sum_{n=1}^{\infty} k_n^3 A_n \sin k_n (x - c_n t) \\ & + \beta \sum_{n=1}^{\infty} k_n A_n \sin k_n (x - c_n t) = 0 .\end{aligned} \tag{5}$$

It is of course obvious that equation (3) is simply a special case of equation (5) in which n is equal to unity only. It is also clear that equation (5) cannot be simplified by dividing all terms by

$$\sum_{n=1}^{\infty} k_n^3 A_n \sin k_n(x - c_n t),$$

as Rossby does in his special case to derive equation (3). In fact, there does not appear to be any simplification of equation (5) which would give the speed of a trough or ridge which is the result of the summation. However it is interesting to note that a particular, but not general, solution to the equation (5) is that each term of the series individually satisfies equation (3). This point will be discussed further in section 6 of this chapter.

2. A Solution for the Velocity of a Trough or Ridge.

Despite the preceding remarks it is possible to derive a formula for the desired velocities of ridges and troughs from purely mathematical reasoning. Inasmuch as we are considering the profile of the 500-mb surface around the entire hemisphere, it seems desirable to redefine the streamline as follows:

$$\psi = \sum_{n=1}^{\infty} A_n \sin(k_n x - c_n t), \tag{6}$$

where k_n is any integer and x and c_n are defined in terms of angular distances and angular velocities. These restrictions insure that the function will be single-valued at any specified longitude. Since the following derivation is purely mathematical and depends only on being able to define the profile by equation (6), no

assumptions whatsoever are necessary as to the physics and dynamics that go to make up the profile.

The longitudes of maximum amplitudes (positive or negative) can be found from equation (6) by differentiating it partially with respect to x, and setting the result equal to zero. This will define the longitudes, x_m, of each trough or ridge implicitly as follows:

$$\sum_{n=1}^{\infty} A_n k_n \cos(k_n x_m - c_n t) = 0 \quad . \qquad (7)$$

Equation (7) also defines the longitudes of any horizontal inflexion points in the profile. However it will be shown later that in practice it is unnecessary to solve this equation, so that the possible horizontal inflexion points can be ignored.

If now the time derivative of equation (7) is taken, it is possible to solve the resultant equation for the instantaneous velocity of each trough and ridge as shown by equation (8).

$$\frac{dx_m}{dt} = V = \sum_{n=1}^{\infty} \Big[\frac{dA_n}{dt} k_n \cos(k_n x_m - c_n t) + A_n k_n t \sin(k_n x_m - c_n t) \frac{dc_n}{dt} + A_n c_n k_n \sin(k_n x_m - c_n t) \Big] \Big[\frac{1}{A_n k_n^2 \sin(k_n x_m - c_n t)} \Big] \quad . \qquad (8)$$

Equation (8) shows some very interesting features. The numerator is divided into three parts, each of which has an effect on the velocity of individual perturbations. The first term shows the effect of changing amplitudes, (amplitude modulation), the second shows the effect of changing speeds, (accelerations), while the

third term is the only one which depends on the speeds of the individual components. This shows that even if each component is stationary, ($C_n \equiv 0$), and remains stationary, ($\frac{dC_n}{dt} \equiv 0$), it is still possible for ridges and troughs to move if the amplitudes of the components change. It is also interesting to note that the speed of any perturbation depends on its location x_m, and thus can be constantly changing even though the speed of each component is constant.

However, since the changes of amplitudes and speeds with time are unknown and probably differ for each component, equation (8) has little more than theoretical interest. If it is assumed that the amplitudes and speeds are not functions of time, the velocity formula (8) reduces to

$$V = \frac{\sum_{n=1}^{\infty} A_n k_n C_n \sin(k_n x_m - C_n t)}{\sum_{n=1}^{\infty} A_n k_n^2 \sin(k_n x_m - C_n t)} , \quad (9)$$

which is perhaps more tractable. Equation (9) shows that even with the restrictive assumptions that amplitude modulations and accelerations are negligible, the movement of individual troughs and ridges is still dependent on the amplitudes as well as the speeds of each component. These ideas can probably best be clarified by a specific example.

3. Example of the Solution of the Velocity Formula.

Even though equations (7) and (9) appear intractable theoretically, they can be solved readily if the number of terms in the series is not too great. In the following hypothetical example, it is assumed, for clarity of exposition and unambiguity

of results, that there are three components of eoual amplitude and that the velocities of the first and second components are zero. With these assumptions, when $\tau =$ zero, equation (7) reduces to

$$\cos X_m + 2\cos 2X_m + 3\cos 3X_m = 0 \quad . \quad (10)$$

By means of trigonemetric identities this is transformed to

$$6\cos^3 X_m + 2\cos^2 X_m - 4\cos X_m - 1 = 0, \quad (11)$$

which yields the following values for cos X_m and X_m.

cos X_m	X_m	(degrees)
.785	38	or 322
- .240	104	or 256
- .875	151	or 209

With the same assumptions used in deriving the values of X_m, eouation (9) reduces to

$$V = \frac{3\,C_3 \sin(3X_m)}{\sin X_m + 4\sin 2X_m + 9\sin(3X_m)} \quad . \quad (12)$$

Taking an hypothetical value of 20 degrees per day for C_3 we find the following values of V for each ridge or trough:

X_m (degrees)	V (degrees per day)
38	4.4
104	6.0
151	10.0
209	12.5
256	5.5
322	3.9

We thus see that even such a simple hypothetical case as the above can produce velocities of ridges and troughs which bear no obvious relation to the velocities of the individual components.

The complete solution of this problem as derived on the Boeing Electronic Analog Computer, as described by Frantz [F], is shown in Plate II. This illustration shows the effect of the movement of sin 3x toward the east at a constant rate of 20 degrees per day, while sin x and sin 2x remain stationary. The complete cycle is shown starting with all components in phase and continuing until they are once again in phase. The varying rates of speed of different troughs and ridges is clearly shown, as well as the changes in speed of the individual perturbations.

Making the same hypotheses as in the previous example, except that the amplitudes of the three components are not assumed equal, results in even more startling conclusions. In this case equation (9) reduces to

$$V = \frac{3 A_3 C_3 \sin (3X_m)}{A_1 \sin X_m + 4 A_2 \sin 2X_m + 9 A_3 \sin 3X_m} . \quad (13)$$

If C_3 is assumed positive, then the numerator and last term of the denominator of equation (13) will have the same sign. Then if the amplitudes are such that

$$|A_1 \sin X_m + 4 A_2 \sin 2X_m| > |9 A_3 \sin 3X_m|, \qquad (14)$$

and the term on the left of the inequality is of an opposite algebraic sign to the term on the right, V will be negative. In other words, with two components stationary and one moving eastward, the resulting perturbation will move westward. If C_3 is negative, then reversing the inequality but maintaining the difference of algebraic sign will also cause the resulting perturbation to move in the opposite direction to C_3. This example has more than theoretical interest since retrograde motion of individual components with a resulting progressive movement of perturbations was actually observed on several occasions during the course of this study.

With the addition of more terms, equations (7), (10), and (11) become increasingly difficult to solve, since the degree of equation (11) is equal to the number of Fourier terms. However in application it would not be necessary to solve these equations for X_m since the continuity chart, already plotted, gives the positions of the ridges and troughs. Therefore, it is only necessary to perform harmonic analysis of the continuity chart, determine the values of amplitudes and speeds of the terms which contribute significantly to the continuity chart, enter these in equation (9) with the longitude of a particular ridge or trough and solve for

its velocity.

It should be noted that in the above derivation the definition of the position of the trough or ridge as the longitude of maximum amplitude differs from the one used in Chapter II.

4. Examples of Harmonic Analysis.

A recent paper of La Seur [8] contained some results of harmonic analysis of the 700-mb continuity charts at 45N during February, 1949. Since it was thought that comparison between the 700-mb and 500-mb surfaces might yield information of value, it was decided to examine the same dates and latitude circle as used by La Seur. Accordingly continuity charts were prepared for February 12 to 22, 1949, both from the Arowa 5-day mean charts and also from the daily charts as given in the Northern Hemisphere Historical Series [16].

The harmonic analysis of these charts was performed mechanically with the Mader-Ott harmonic analyser described by Meyer zur Capellen [11]. Some of the limitations of this instrument are discussed in the Appendix. The primary results of these analyses are presented in Tables 8 and 9. In these tables amplitudes are given in 10's of feet and the displacement angle d, is the longitude of the ascending node. In the case of waves of number 2 or greater this displacement angle could equally well be placed at any integer number of wave lengths from the figures given in the table. For example, the fourth term of February 12 of the daily charts could equally well be assigned a displacement angle of 87W, 3E, or 93E instead of 177W as given in the table, since the four curves resulting from these different displacements are indistinguishable. The tabular values were determined in the following manner. For the first day analyzed, any

convenient one of the possible values was assigned. The subsequent daily values were then assigned in such a way that the change of displacement angle would never be greater than one-half wave length. In this way it is possible to determine both a speed and a direction for each component. The speed was defined as the change of the displacement angle in 24 hours. If the change of displacement angle was exactly one-half wave length the direction of movement is ambiguous. Luckily this occurred very rarely in the charts examined. When this occurred a direction of movement was assigned arbitrarily to agree with the previous movement.

TABLE 8

Amplitudes and Displacements of Fourier Components of Daily Continuity Charts for February 1949

No.	Wave Length		Date 12	13	14	15	16	17	18	19	20	21	22
1	360	A	22	32	29	19	59	39	41	48	58	69	65
		d	170E	166E	169W	118W	104W	100W	109W	111W	110W	107W	110W
2	180	A	31	44	32	39	43	30	27	29	16	11	7
		d	96E	96E	106E	117E	146E	160E	176W	179W	173W	128W	173E
3	120	A	46	41	48	46	47	41	30	20	18	20	26
		d	134E	170E	180	179E	173W	153W	113W	132W	96W	97W	90W
4	90	A	14	32	17	31	28	17	14	18	21	7	2
		d	177W	169W	147E	147E	157E	138E	112E	124E	141E	164E	178E
5	72	A	24	22	13	7	6	5	3	7	14	14	19
		d	127E	139E	125E	129E	151E	174W	178W	173E	173E	140E	134E
6	60	A	38	26	15	20	20	24	19	12	6	9	6
		d	145E	134E	158E	128E	111E	105E	114E	102E	83E	72E	72E
7	$51\frac{3}{7}$	A	14	12	6	28	28	18	35	22	26	12	9
		d	167E	171E	177E	179E	168W	180	156E	156E	151E	140E	126E
8	45	A	8	13	8	9	14	35	7	9	14	10	22
		d	159E	175E	157E	157E	146E	134E	120E	122E	129E	116E	129E
9	40	A	7	5	3	9	6	10	7	17	9	5	21
		d	148E	151E	158E	162E	143E	124E	130E	111E	122E	123E	127E
10	36	A	15	15	2	10	15	8	16	12	6	6	4
		d	136E	152E	156E	159E	171E	178W	174W	161W	146W	145W	146W

TABLE 9

Amplitudes and Displacements of Fourier Components
of 5-Day Mean Continuity Charts for February, 1949

Wave No.	Wave Length		Date 12	13	14	15	16	17	18	19	20	21	22
1	360	A	21	22	24	25	33	38	45	49	55	60	63
		d	178W	178W	157W	137W	116W	107W	106W	104W	108W	108W	107W
2	180	A	31	34	36	34	32	31	31	24	21	12	4
		d	100E	103E	112E	124E	146E	158E	170E	176W	167W	160W	169W
3	120	A	45	34	46	42	41	32	28	24	21	22	24
		d	171W	176W	180	176E	161W	149W	138W	119W	101W	97W	151W
4	90	A	16	18	21	20	22	19	19	15	13	9	4
		d	128E	124E	130E	133E	147E	141E	142E	138E	126E	125E	81E
5	72	A	11	13	13	7	4	5	4	5	7	6	8
		d	121E	131E	142E	171E	171E	179W	157W	179W	155E	171E	168E
6	60	A	16	17	12	4	4	2	8	7	5	0	2
		d	172E	167E	137E	150E	150E	133E	143E	160E	140E	---	128E
7	$51\frac{3}{7}$	A	8	6	13	11	15	17	17	9	11	11	7
		d	154E	133E	140E	115E	112E	112E	115E	93E	94E	88E	76E
8	45	A	9	11	8	3	5	11	12	12	6	8	4
		d	157E	173E	166E	150E	156E	173E	156E	162E	144E	136E	136E
9	40	A	2	6	2	2	4	7	4	2	5	7	2
		d	168E	150E	150E	148E	138E	126E	145E	145E	126E	132E	132E
10	36	A	5	4	4	5	2	1	5	2	1	6	4
		d	133E	120E	109E	104E	106E	88E	97E	100E	95E	107E	118E

Inspection of Tables 8 and 9 shows that the assumption that amplitudes and speeds are not functions of time, made in deriving equation (9), is not justified. The irregularity of the movements of various components is shown by Figures 1 and 2. These graphs show the displacement longitude of each of the Fourier components. The wave number is shown at the left and the displacement angles are numbered consecutively rather than by dates so that 1 represents February 12, etc.

Figure 1 shows the longitude of the displacement angles of the components of the daily continuity charts, and Figure 2 shows the displacement angles for the 5-day mean charts. It is shown in the Appendix that when the amplitude is less than 161 feet, the displacement angle is not reliable within 5 degrees. These unreliable displacements are shown encircled on Figures 1 and 2. If these unreliable displacements are disregarded, it is believed that some regularity of movement can be detected in the first few components. Thus the first component of both the daily and 5-day mean charts tend to become stationary near 110W. This agrees with La Seur [8], who stated that the first harmonic term showed the eccentricity of the circulation with the circulation pole usually near 180 degrees longitude. The second and third harmonic terms also show a fairly regular movement.

5. Comparison of Fourier Terms Between Daily and 5-Day Mean Charts.

A comparison of the amplitudes of the Fourier components shown on the daily chart and the 5-day mean chart centered on the same date should reveal which components are damped out in the 5-day averaging process. Accordingly the correlation coefficients were computed for each term for this 11-day sample. The results are shown in Table 10.

For only 11 cases a correlation coefficient must be greater than .70 for 95% confidence and greater than .58 for 90% confidence. It seems justified to conclude that the components with wave number 6 or less on the daily charts are related to those on the 5-day mean charts. Putting this another way, one might say that all waves whose lengths ~~is~~ are less than 60 degrees of longitude are damped out by

Figure 1

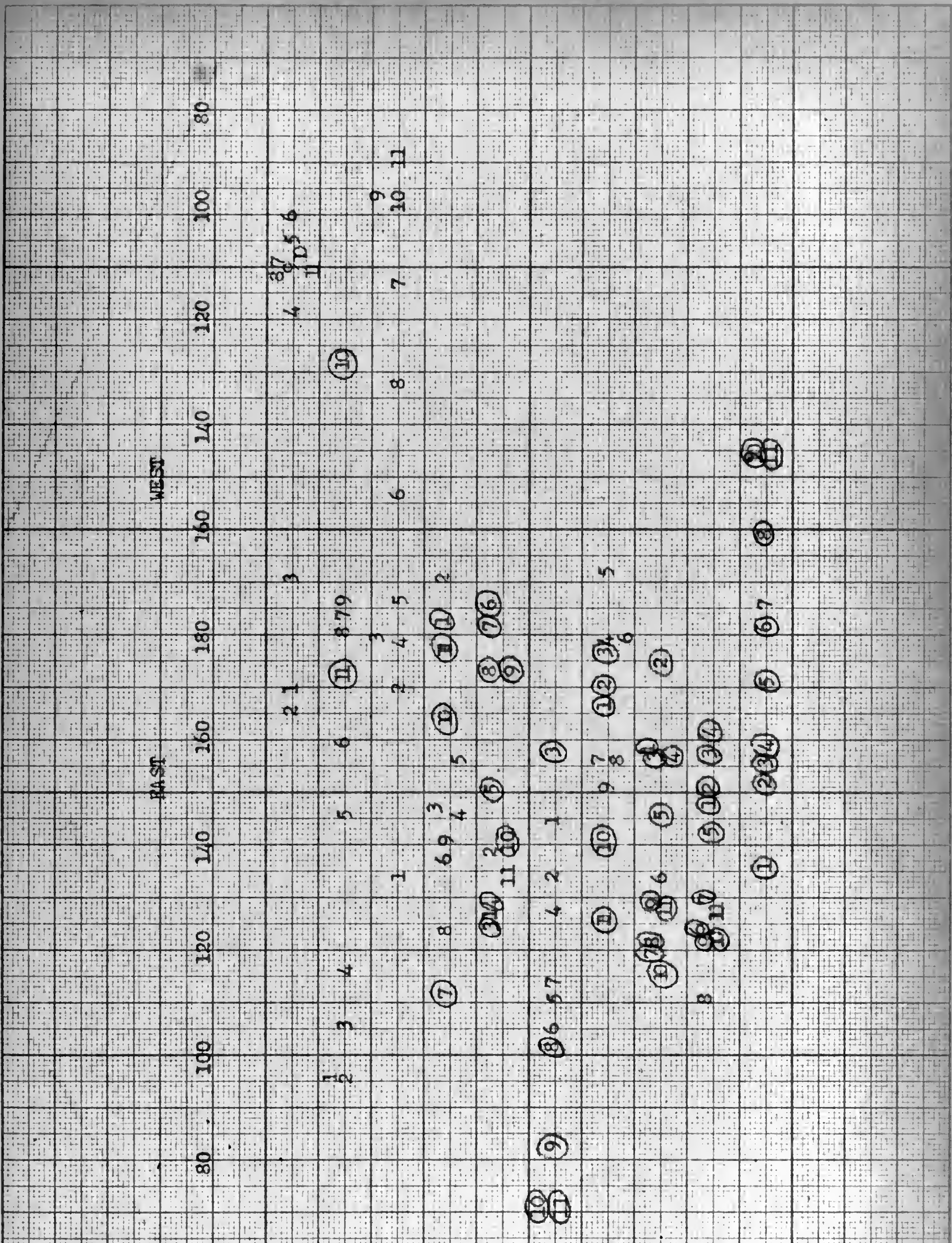


Figure 2

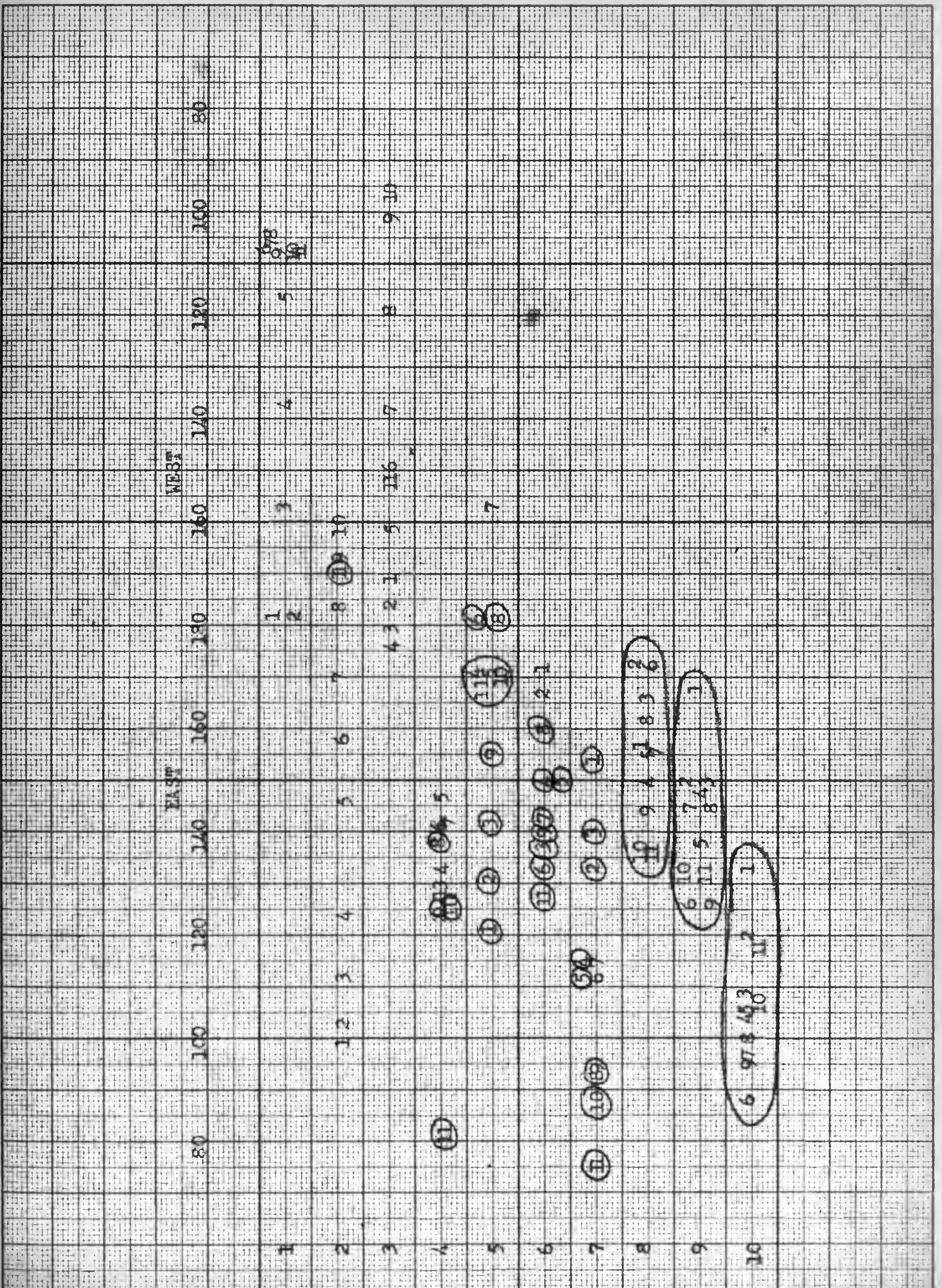


TABLE 10

Correlation Coefficients Between Amplitudes of Fourier Terms of Daily and 5-Day Mean Charts

Wave Number	Correlation Coefficient
1	.923
2	.954
3	.855
4	.796
5	.726
6	.660
7	.489
8	.010
9	-.288
10	-.236

the 5-day averaging process. As mentioned earlier, the damping out of "short" waves was one of the reasons for developing the 5-day mean charts, so that this result is not surprising. It does, however, provide a quantitative determination of the damping. This admittedly small sample would indicate that all waves less than 60 degrees in length are short waves. However, increasing the size of the sample might make the correlation for wave number 7 significant in which case waves shorter than 51 3/7 degrees would be the short waves. It seems very doubtful that the correlations for wave numbers 8, 9, or 10 would ever approach significance.

6. The Rossby Wave Equation for Individual Components.

It was mentioned in section 1 of this chapter that a particular but not general solution to the Rossby wave equation, when the

streamline is represented by a Fourier series, is that each term individually satisfy equation (3). Using the 24-hour change of displacement angle as a speed, it is possible to solve the Rossby equation for $\bar{U}$, the mean zonal wind. If each term taken separately gave the same value of $\bar{U}$, it would be evident that the particular solution has more than theoretical interest. This computation was done on the second through sixth terms of Table 8 and the results are shown in Table 11.

TABLE 11

Mean Zonal Wind (in meters per second) Computed from the Velocities of Each Component of the Harmonic Analysis

Wave Number	Date 13	14	15	16	17	18	19	20	21	22
2	82	91	91	107	85	94	80	87	124	32
3.	67	45	29	43	53	70	21	67	36	36
4.	27	-17	21	29	37	-2	31	35	40	34
5	23	-1	17	32	43	10	6	13	-15	8
6	4	29	35	-5	4	13	-1	-7	0	9

Reading down the columns of Table 11 for any one day it can be seen that there is rarely agreement even in order of magnitude. Evidently the individual terms do not satisfy the Rossby equation.

7. A 3-Term Approximation to the Continuity Chart.

As mentioned earlier, the lower harmonics of the analyses seem to have some regularity of movement as shown on Figures 1 and 2. If the amplitudes of each component are added, the resultant sum represents the maximum amplitude if all harmonics were in phase. The percentage of each component's contribution to this maximum amplitude

TABLE 12

Percentage Contribution of Each Fourier Component of Daily Continuity Charts to the Maximum Amplitude

Wave Number	Date 12	13	14	15	16	17	18	19	20	21	22	Avg.
1	10.1	13.3	16.8	8.8	22.3	17.2	20.6	24.8	30.7	42.3	37.0	22.2
2	14.0	18.4	18.3	18.0	16.0	13.3	13.4	14.8	8.4	6.9	1.0	13.0
3	21.1	16.8	27.6	21.2	17.9	18.1	15.0	10.0	9.4	12.4	14.7	16.7
4	6.4	13.1	10.1	14.1	10.4	7.6	7.3	9.5	11.0	4.2	1.4	7.7
5	10.7	8.9	7.7	3.3	2.3	2.1	1.5	3.5	7.7	8.5	10.9	5.2
6	17.2	10.8	8.5	9.1	7.5	10.4	9.6	6.4	3.3	5.8	3.5	9.3
7	6.5	4.9	3.4	13.1	10.5	7.7	17.5	11.1	14.0	7.4	5.3	9.2
8	3.9	5.5	4.8	4.0	5.3	15.4	3.7	4.8	7.7	6.3	12.4	6.7
9	3.2	1.9	1.8	4.1	2.2	4.5	3.5	8.8	4.9	2.8	11.8	4.5
10	7.0	6.4	1.1	4.5	5.7	3.7	8.0	6.1	3.0	3.6	2.0	4.7

TABLE 13

Percentage Contribution of Each Fourier Component of 5-Day Mean Continuity Chart to the Maximum Amplitude

Wave Number	Date 12	13	14	15	16	17	18	19	20	21	22	Avg.
1	12.8	13.3	13.4	16.3	20.3	23.3	26.0	32.9	37.9	42.6	51.6	26.4
2	18.9	20.6	20.1	22.2	19.8	19.0	17.9	16.1	14.5	8.5	3.3	17.2
3	27.4	20.6	25.7	27.5	25.3	19.6	16.2	16.1	14.5	15.6	19.7	20.8
4	9.8	10.9	11.7	13.1	13.6	11.7	11.0	10.1	9.0	6.4	3.3	10.1
5	6.7	7.9	7.3	4.6	2.5	3.1	2.3	3.4	4.8	4.3	6.6	4.9
6	10.0	10.3	6.7	2.6	2.5	1.2	4.6	4.7	3.4	0	1.6	4.3
7	5.0	3.6	7.3	7.2	9.3	10.4	9.8	6.0	7.6	7.8	5.7	7.3
8	5.5	6.7	4.5	2.0	3.0	6.7	6.9	8.1	4.1	5.7	3.3	5.1
9	1.2	3.6	1.1	1.3	2.5	4.3	2.3	1.3	3.4	5.0	1.6	2.5
10	3.0	2.4	2.2	3.3	1.2	0.6	2.9	1.3	0.7	4.3	3.3	2.3

is considered a good measure of its relative importance. These percentages are shown in Tables 12 and 13. It is evident from these tables that the lower harmonics contribute between 45 per cent and 63 per cent of the total amplitude, while the first six contribute between 67 per cent and 89 per cent. For the 5-day mean charts, the contribution of the lower harmonics is generally larger as might be expected, ranging from 55 per cent to 75 per cent for the first three harmonics and from 77 per cent to 86 per cent for the first six. Using a slightly different approach, La Seur [8] concluded that the first six harmonics gave an adequate representation of the continuity charts at 700-mb. Using 36 distinct points, he concluded that over 90 per cent of the variance of height was accounted for by the first six harmonics.

In order to test the adequacy of the first three harmonics to represent the continuity chart, the amplitudes and displacements shown in Tables 8 and 9 were added back together on the Boeing Analog Computer and compared with the original charts. It was found that on the daily charts the 3-term approximation was best on February 14 and worst on February 19. On the 5-day mean charts the 3-term approximation was best on February 14 also, but worst on February 17. It was then decided to add in harmonics 4 and 6, and finally harmonics 7, 8, and 9. Harmonics 5 and 10 were not added because of mechanical difficulties with the computer. The results of these syntheses are shown in Figures 3, 4, 5, and 6. The upper solid curve in each figure is the result of adding the first three harmonics, the middle solid curve, the result of adding harmonics 4 and 6 to the first three, and the bottom solid

Figure 3

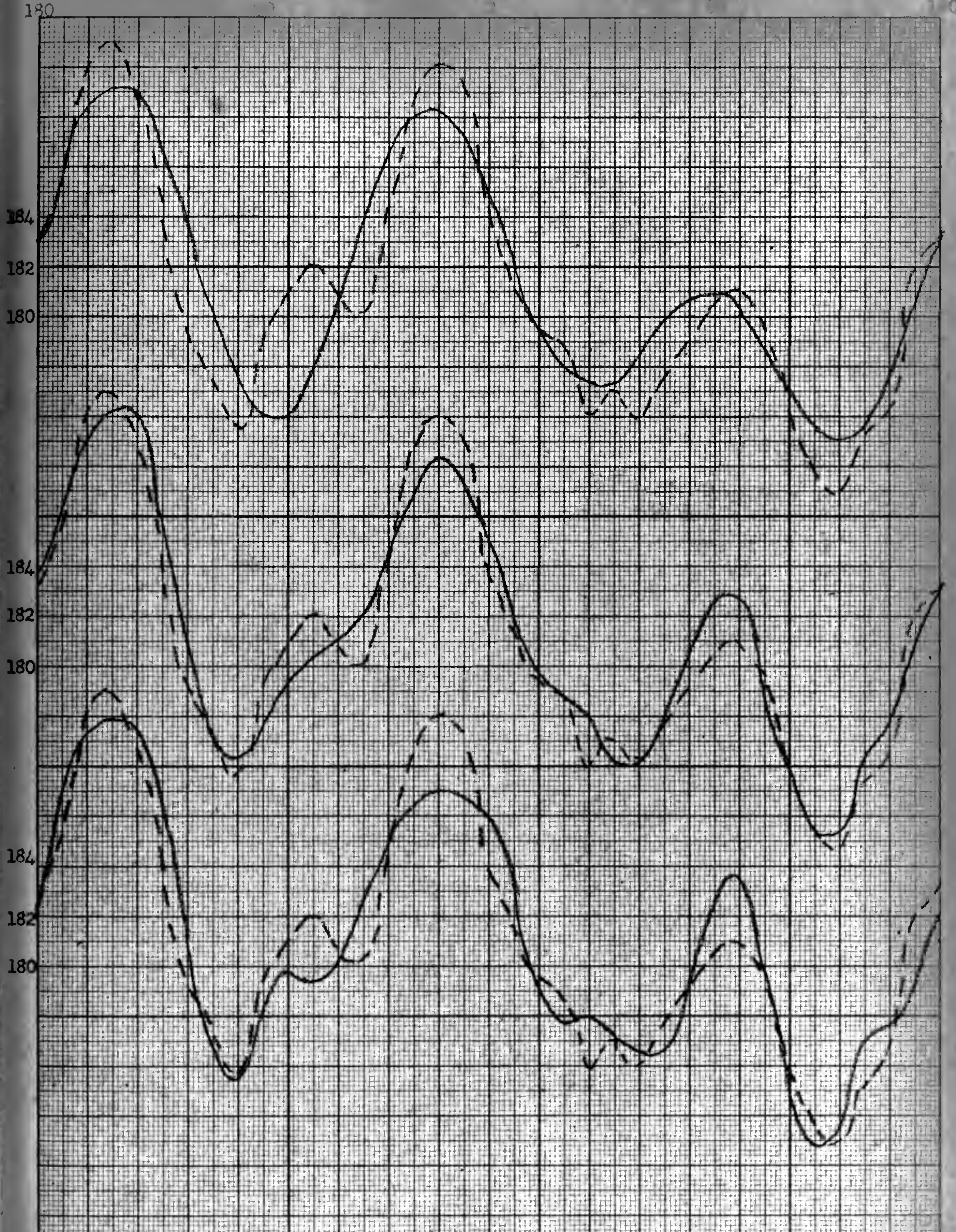


Figure 4

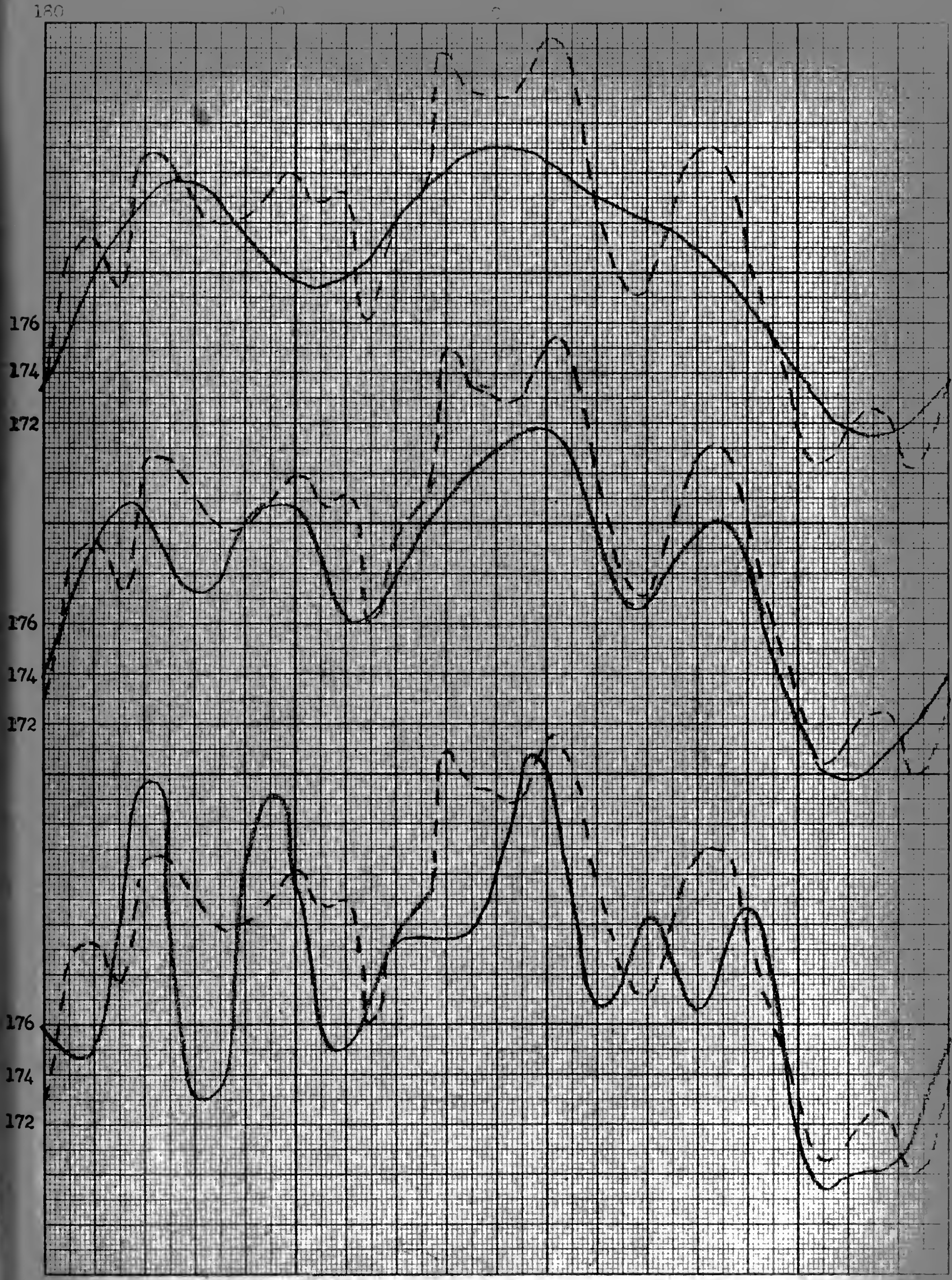


Figure 5

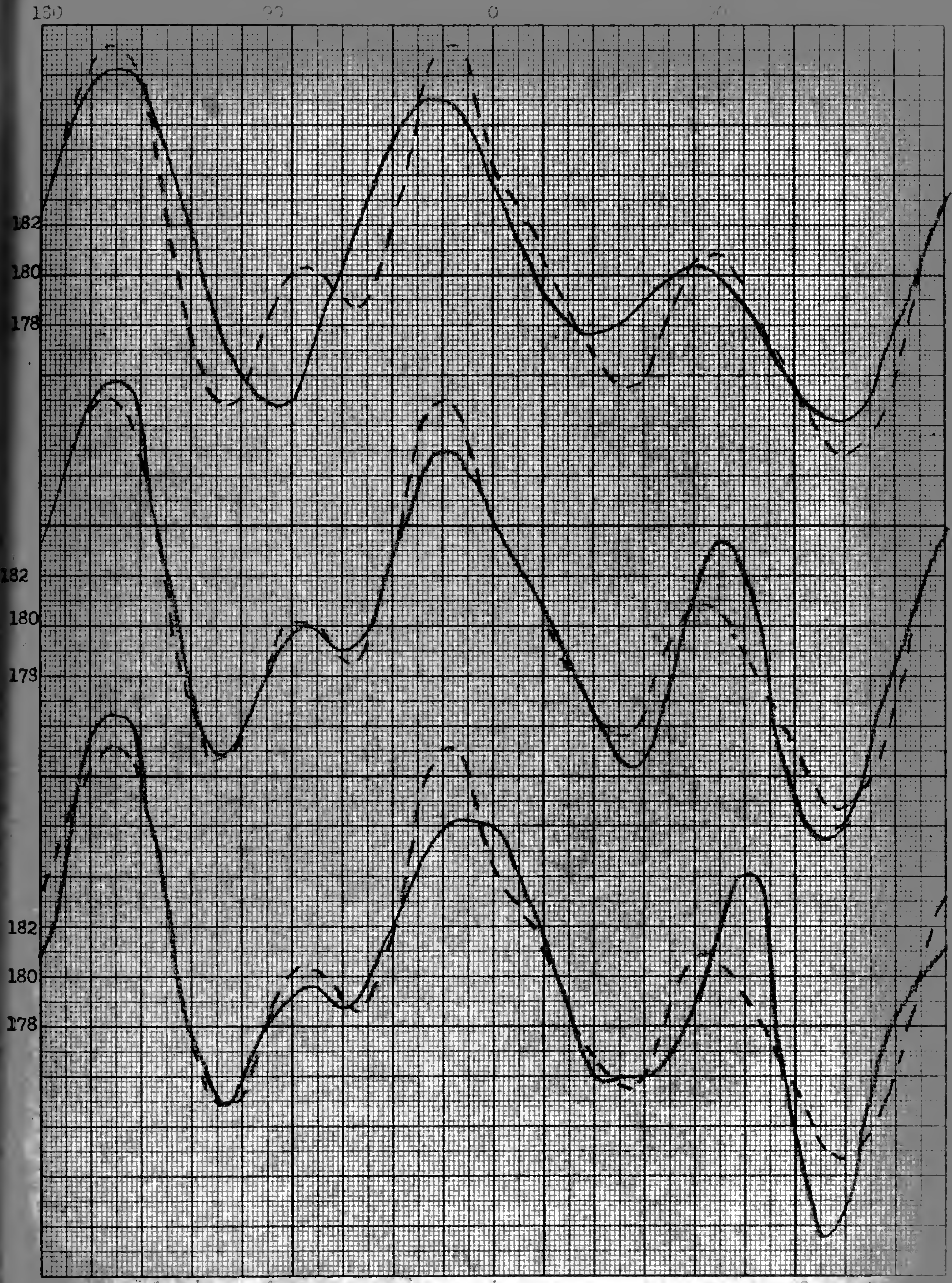
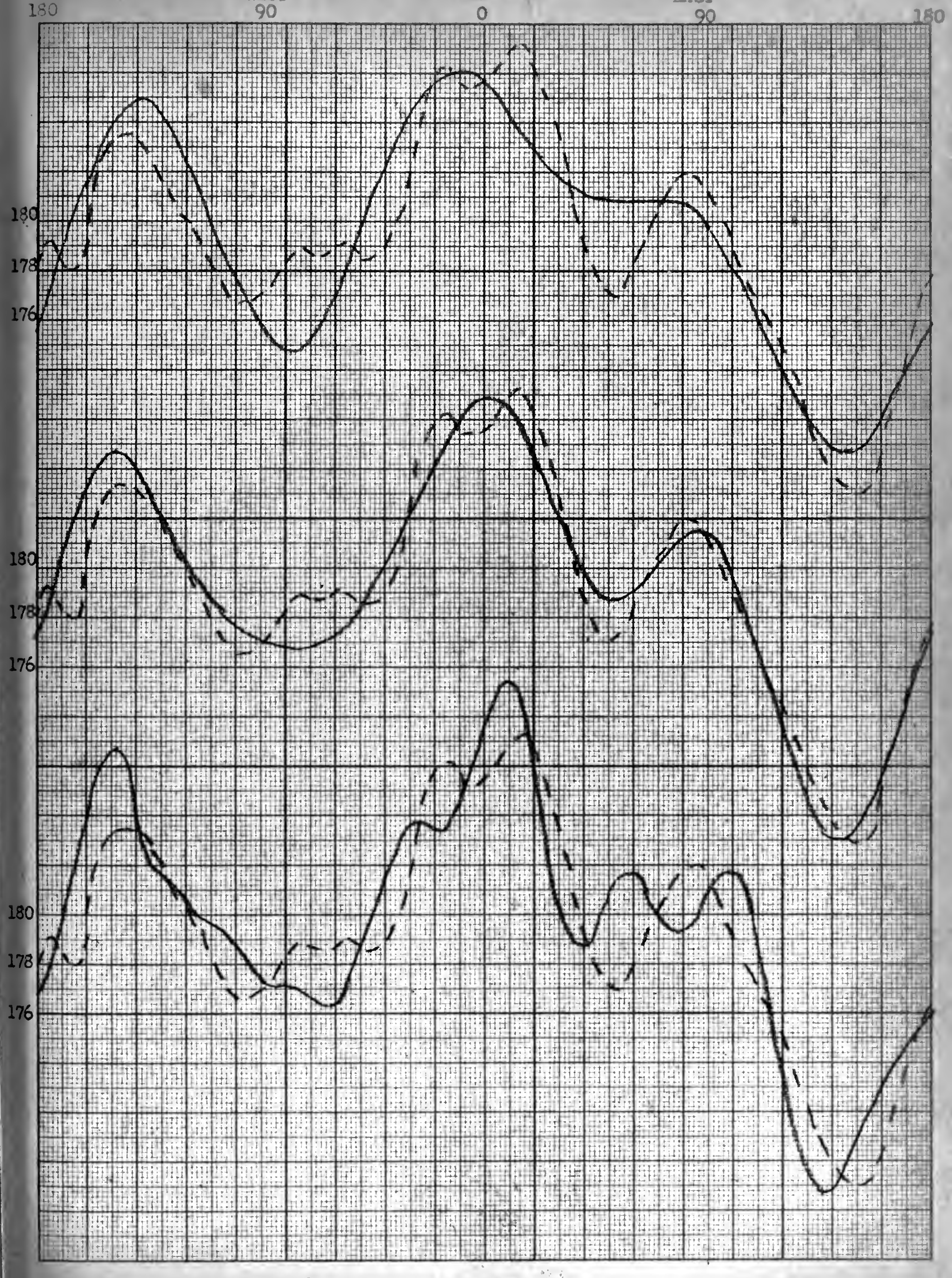


Figure 6



curve the result of adding harmonics 7, 8, and 9 to the other five. For comparison purposes the original curve from the continuity charts is shown as a dashed line in each figure. Figures 3 and 4 show the curves for the February 14 and February 19 daily charts respectively, and Figures 5 and 6 for the 5-day mean charts of February 14 and February 17 respectively. It is clear that while the addition of the fourth and sixth harmonics results in some improvement to the approximations, the addition of the seventh, eighth, and ninth add very little. In fact, in some cases they seem to make the approximation worse. This may be due at least in part to the inadequacies of the analog computer as discussed in the appendix. It should also be noted that even the worst-approximated mean chart is fairly well represented by five terms.

8. Harmonic Analysis as a Forecasting Tool.

Tables 8 and 9 and Figures 1 and 2 indicate that there is a certain amount of continuity to the terms of the Fourier components, at least for the lower harmonics, while Figures 3, 4, 5, and 6 show that three or at most six terms of the Fourier series will show the major trough and ridge positions. Equation (9) provides a means of forecasting the movements of troughs and ridges if the amplitude and velocity of each harmonic term does not vary with time. If they do change and the changes can be forecast even by persistence, mean values could be used in equation (9) or machine computation could provide a rapid solution.

The question remains as to the continuity of the Fourier terms themselves. It is felt by the author that some of the apparent discontinuity may be due to the basic assumption made in the use and

preparation of continuity charts mentioned in the introduction, that the atmospheric flow is strictly zonal. As pointed out by La Seur [8] and confirmed in the present study, this assumption is frequently erroneous. Tables 8 and 9 and Figures 1 and 2 show that the continuity of amplitudes and speeds of the harmonic terms is better for the 5-day mean charts than for the daily charts. This is no doubt the result of averaging out much of the meridional flow. It is believed that even better results than shown on the 5-day time-mean charts would result from the preparation of daily space-mean charts in which the heights at several points around each longitude-latitude intersection were averaged and the resultant plotted on a continuity chart. To some extent this is the method originally used by Cressman [4], although he averaged the height only along the latitude circles, not longitudinally as well.

It is understood that Project Arowa is at present engaged in preparing space-mean charts of the type suggested. When these are available it should be possible to test this hypothesis.

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APPENDIX

THE ACCURACY OF MACHINE COMPUTATIONS

1. The Mader-Ott Harmonic Analyser.

The Mader-Ott harmonic analyser consists of two planimeters connected to a stylus with a linkage so that by tracing any function with the stylus the readings of the planimeters give the amplitudes S_n and C_n of a Fourier series in the form

$$G(x,t) = \sum_{n=1}^{\infty} S_n \sin nx + C_n \cos nx \quad . \tag{A.1}$$

By using different linkages it is possible to find the amplitude of each term up to and including the 33rd.

For meteorological purposes a series of the form

$$G(x,t) = \sum_{n=1}^{\infty} A_n \sin(nx - \alpha_n) \quad , \tag{A.2}$$

is more useful. To convert equation (A.1) to equation (A.2) form the identity

$$\sum_{n=1}^{\infty} A_n \sin(nx - \alpha_n) \equiv \sum_{n=1}^{\infty} S_n \sin nx + C_n \cos nx \quad , \tag{A.3}$$

expand the left hand side by trigonometric identities, and equate the coefficients of $\sin nx$ and $\cos nx$. It follows that

$$A_n \cos \alpha_n = S_n \quad , \tag{A.4a}$$

$$A_n \sin \alpha_n = - C_n \quad . \tag{A.4b}$$

and hence that

$$A_n = \sqrt{S_n^2 + C_n^2} \quad , \qquad (A.5)$$

and

$$d_n = \cos^{-1} \frac{S_n}{A_n} = \sin^{-1} - \frac{C_n}{A_n} \quad . \qquad (A.6)$$

The scale of the instrument is such that an amplitude of one cm gives a reading of 100 on the planimeters. It was determined by repeated trials of the instrument that the operator error in tracing a curve was never greater than 5 units on the planimeter. Using the well-known formula for the error of a function, it follows from equation (A.5) that

$$\Delta A_n = \frac{S_n}{A_n} \Delta S_n + \frac{C_n}{A_n} \Delta C_n \quad , \qquad (A.7)$$

but

$$|\Delta S_n| = |\Delta C_n| = 5 \quad ,$$

as determined by repeated trials, and

$$\frac{S_n}{A_n} = \cos d_n \quad , \quad \frac{C_n}{A_n} = -\sin d_n \quad .$$

Since the maximum possible value for $\cos d_n - \sin d_n = \sqrt{2}$ it follows that

$$\Delta A_{n\,max} = 5\sqrt{2} \quad . \qquad (A.8)$$

Since the continuity charts were plotted with a vertical scale of one cm equal to 100 feet, the maximum possible error in the amplitude determinations was $5\sqrt{2}$ ft. Since upper-air heights are only reported to the nearest 10 feet, the accuracy of the analyser is greater than the data justify as far as the amplitudes of the Fourier components are concerned.

Using the error formula on equation (A.4a), it follows that

$$\Delta d_n = \frac{1}{\sqrt{1-\left(\frac{S_n}{A_n}\right)^2}}\,\frac{S_n}{A_n^2}\,\Delta A_n - \frac{1}{\sqrt{1-\left(\frac{S_n}{A_n}\right)^2}}\,\frac{\Delta S_n}{A_n} \quad . \qquad (A.9)$$

Again $\frac{S_n}{A_n} = \cos d_n$, and if $\frac{\pi}{4} \leq d_n \leq \frac{\pi}{2}$,

i.e. $\sin d_n > \cos d_n$ or $C_n > S_n$,

then the maximum possible value of $\cos d_n$ is $\frac{1}{\sqrt{2}}$. Substituting this value, as well as those for ΔA_n and ΔS_n previously determined, it follows that

$$\Delta d_{n\,max} = \frac{10\sqrt{2}}{A} \text{ radians or } \frac{805}{A} \text{ degrees.} \qquad (A.10)$$

An identical development from equation (A. 4b) leads to an identical formula for $\Delta d_{n\,max}$ when $0 \leq d \leq \frac{\pi}{4}$. It is thus clear that for maximum accuracy in the determination of

d_n, equation (A.4a) is used if $C_n > S_n$ and (A.4b) if $S_n > C_n$.

From equation (A.10) it follows that for the possible error in the displacement angle to be less than 5 degrees A_n must be greater than 161, and for the error to be less than 10 degrees A_n must be greater than 81. On Figures 1 and 2, which show these displacement angles, those which may be in error by greater than 5 degrees are encircled.

2. The Boeing Electronic Analogue Computer.

The Boeing Analogue Computer carries a guarantee by the manufacturer to be accurate within 20 per cent. In practice however, it is possible to get results considerable better or worse than this depending on a multiplicity of factors such as the steadiness of the power source, the age of the vacuum tubes, the accuracy with which capacitances and resistances can be determined, and the complexity of the problem. The accuracy which was achieved on the present problem is illustrated by Figure 7. This illustration shows three curves representing $y = \sin x$, $y = \sin 2x$, and $y = \sin 3x$. The amplitudes are supposed to be equal. There is some random "noise" evident, particularly in the curves for sin 2x and sin 3x, within the first 40 degrees. It is also shown that the curves do not all cross the x-axis at exactly 180 and 360 degrees as they should. This error is less than 2 mm in 180 or slightly greater than 1 per cent. More serious is the damping, evident particularly in the curve for sin 3x. This amounts to a 15 per cent error on the 3rd cycle of this curve. However, even this is not considered to be too serious an error, since the computer was only used to determine the general shape of the 3-term approximations. On the scale used

this error amounts to about 100 feet and the 3-term approximations may be in error by this amount, if the amplitude of the third component is as much as 670.

For the higher terms, the error would be more serious. For instance, on the ninth cycle the sin 9x curve might be in error by as much as 45 per cent if the damping is linear. This is an additional reason for confining attention to the first six terms at most of the Fourier series, if this type of computer is to be used.

Figure 7

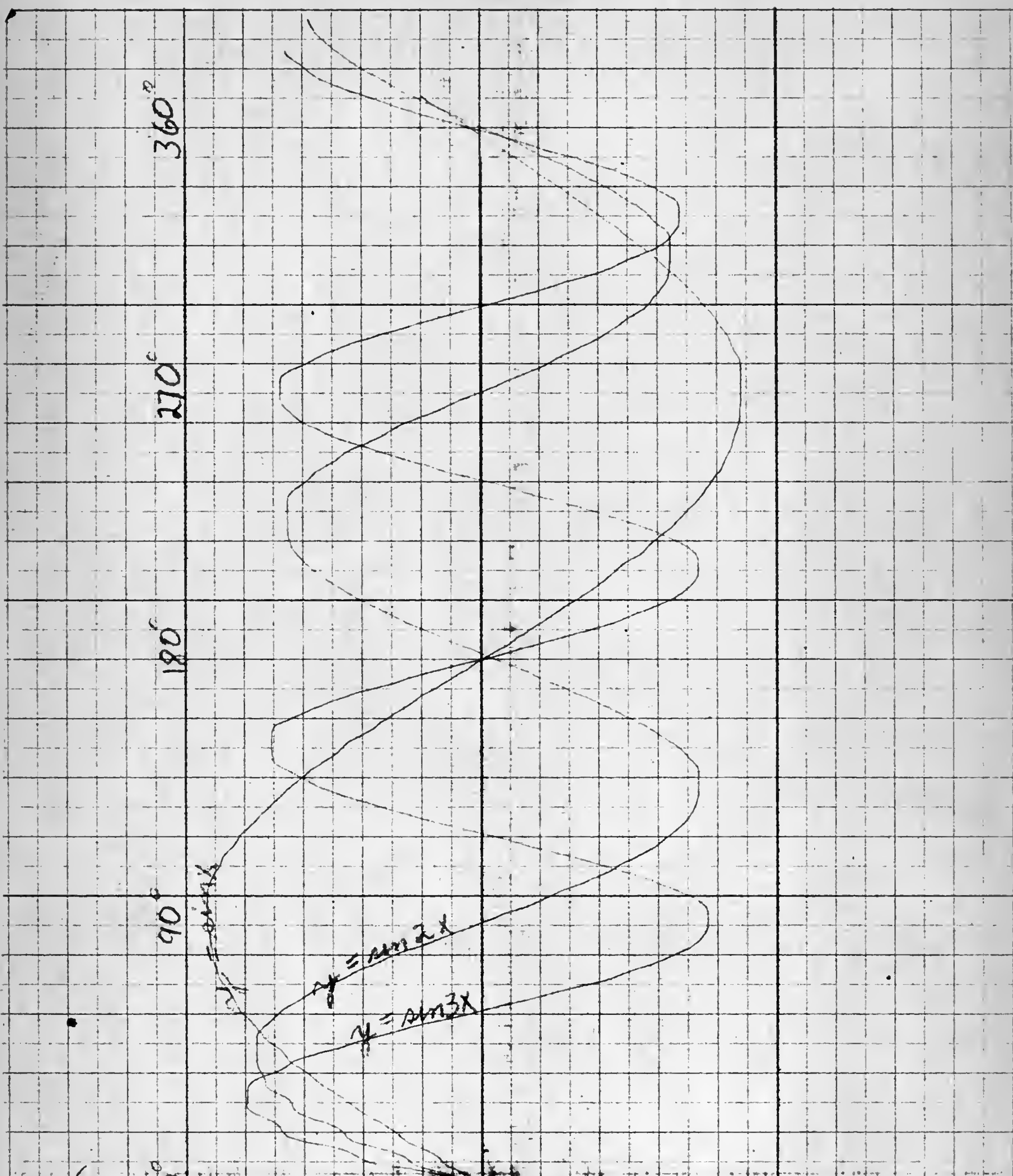
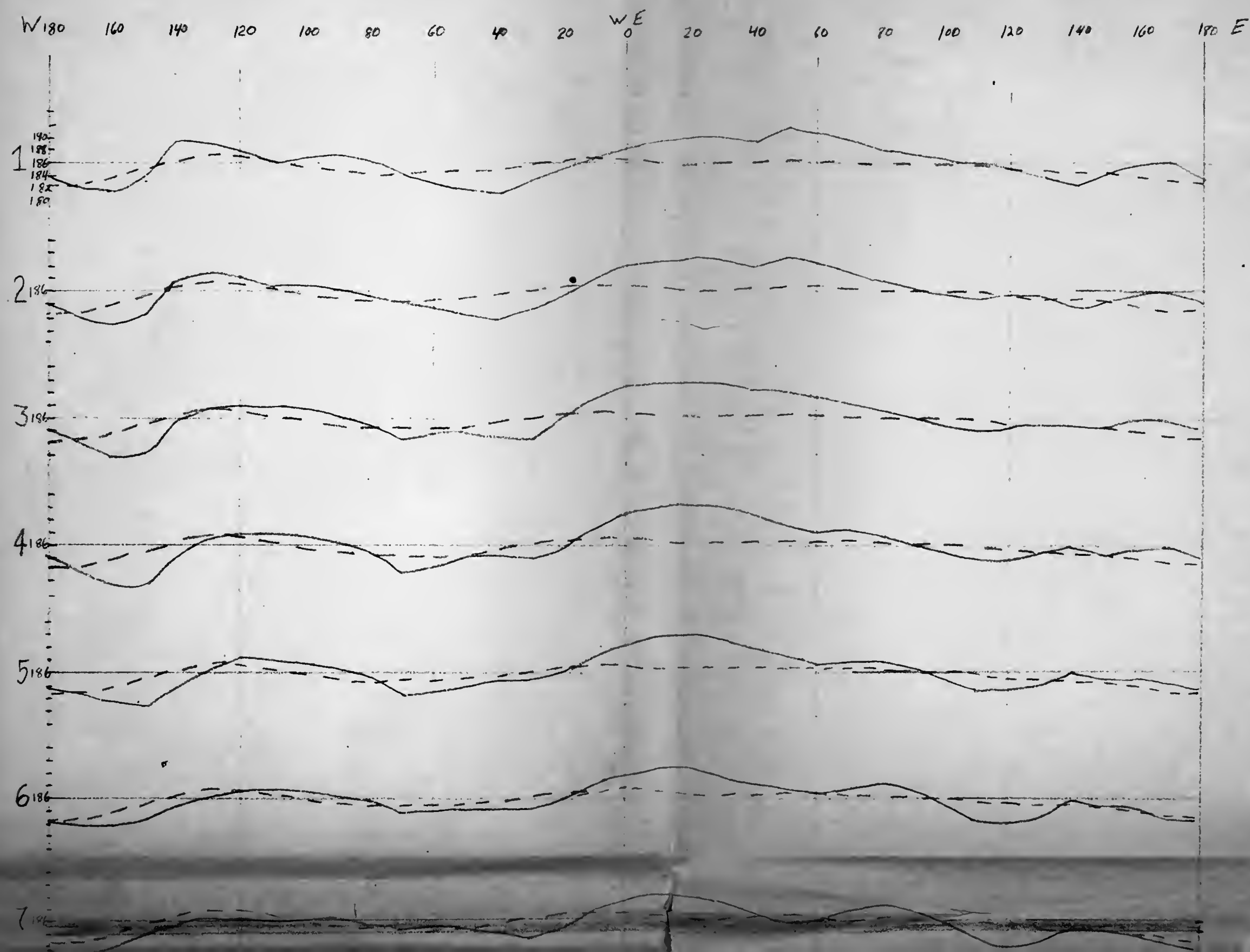
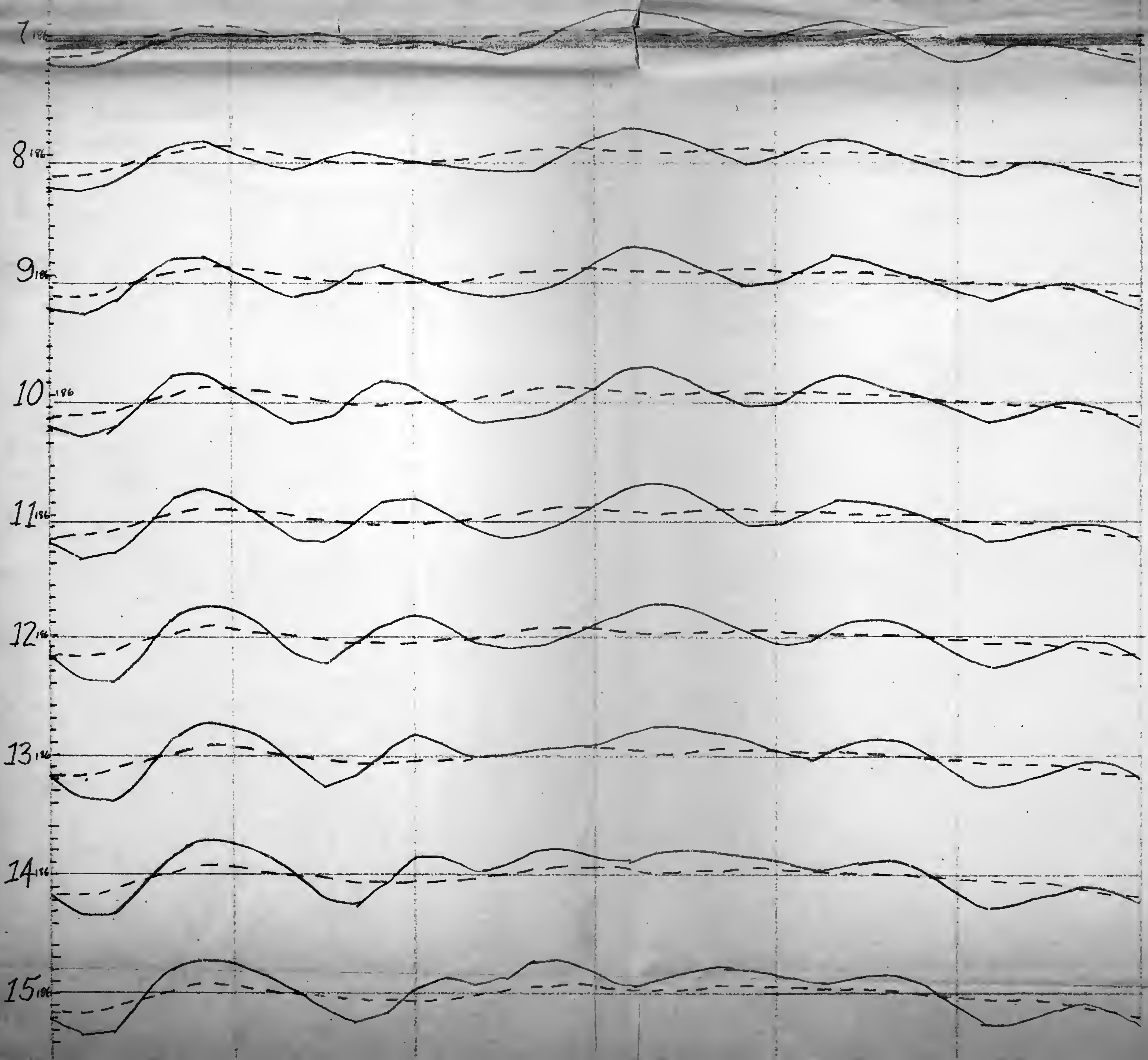


PLATE I

CONTINUITY CHART 500 MB - 5 DAY MEANS AT 50°N LAT

SEPT 1-30 1951





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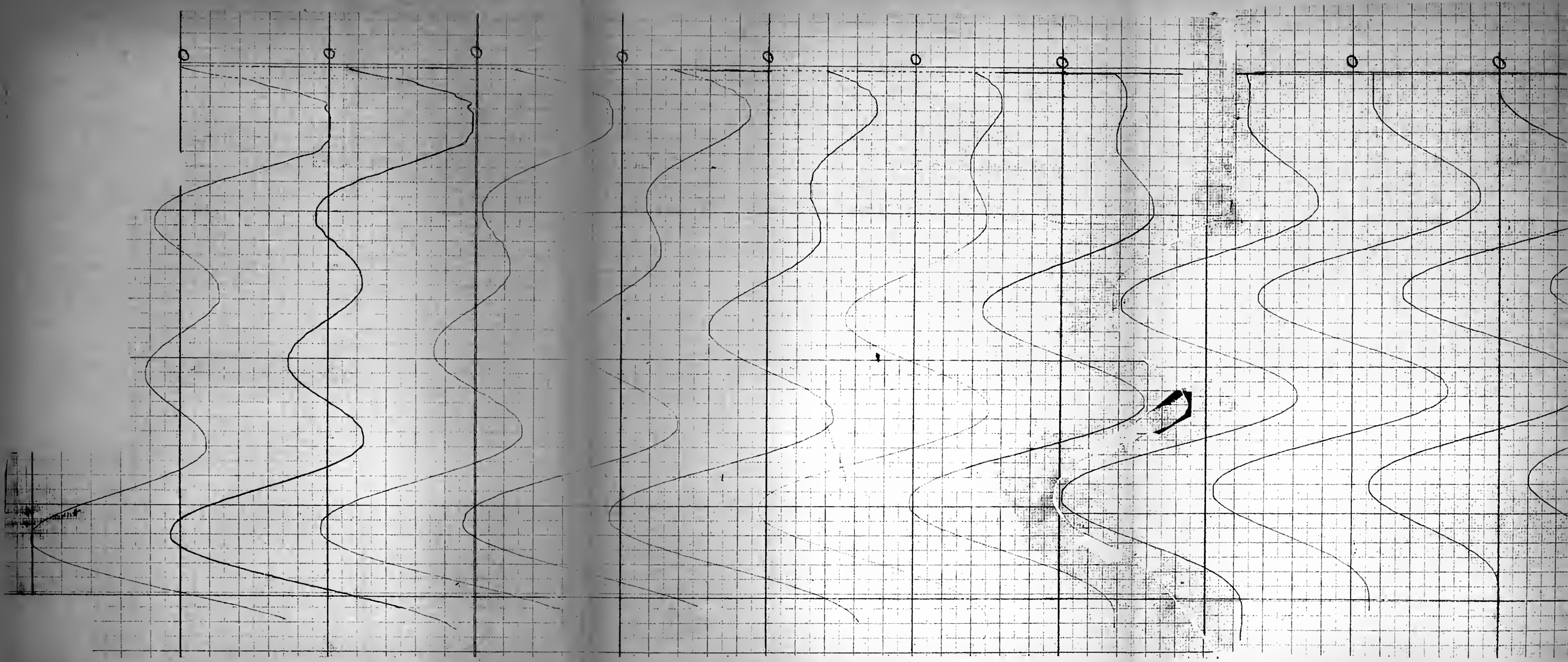
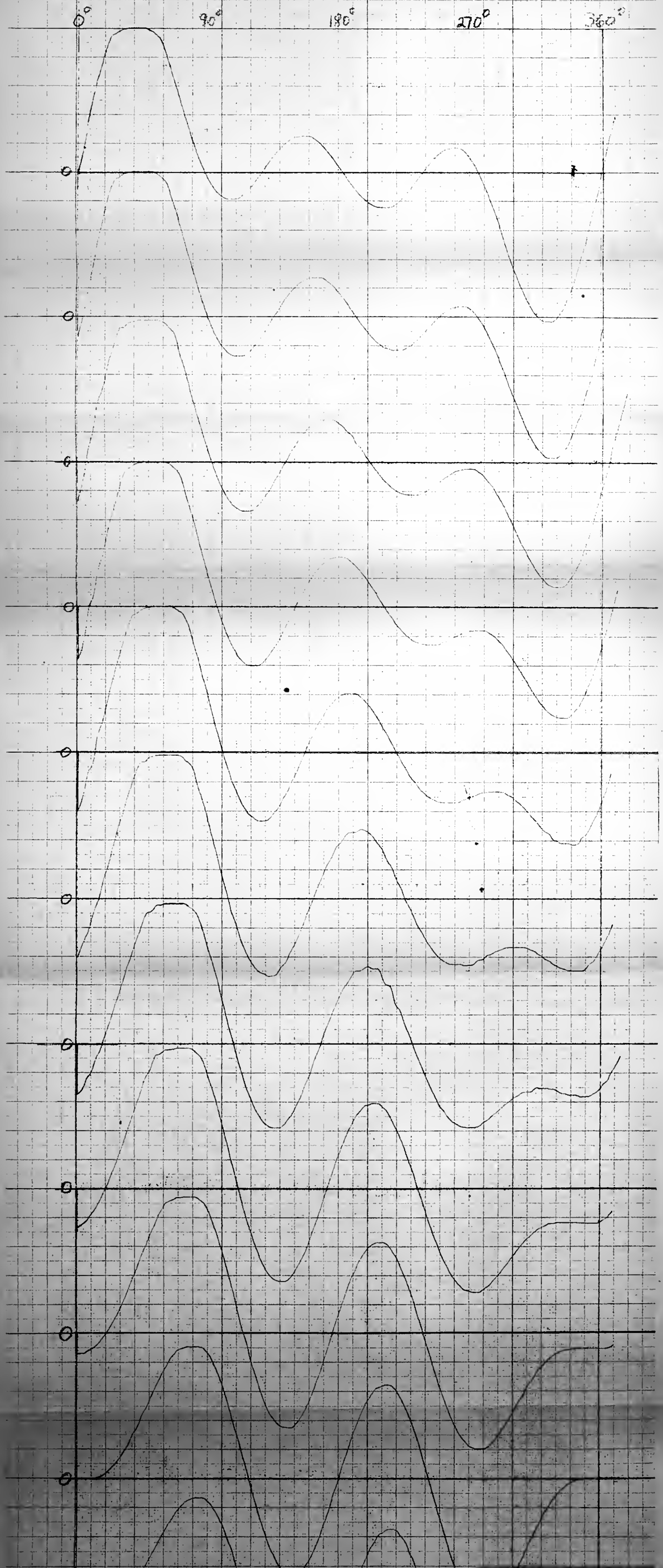


PLATE II



Basquin
The movements of troughs and ridges on the 500-mb continuity chart.

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The movements of troughs and ridges on the 500-mb continuity chart.